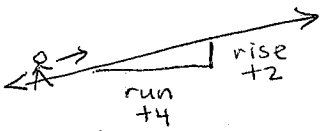
Math 45 SSM 2/e 3.3 Slope p.1

Objectives:
1) Positive & negative slopes and slopes of vertical and horizontal lines.
2) Use the slope formula to find the slope of a line.
3) Graph a line using a point on the line and the slope.
4) Applications of slope

Describe the differences among these lines/graphs:

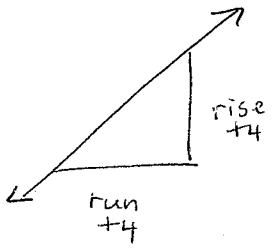
(1)

(A)



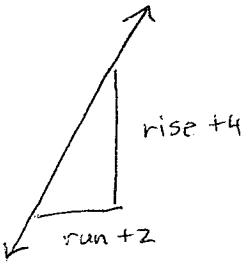
uphill
not very steep

(B)



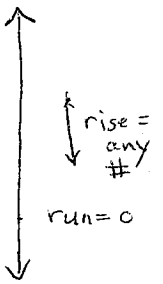
uphill
moderately steep

(C)



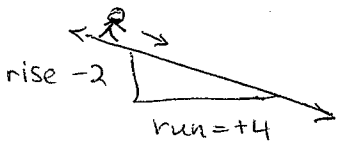
uphill
very steep

(D)



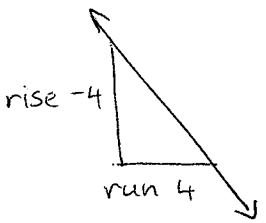
impossibly
uphill

(C)



downhill
not very steep

(D)

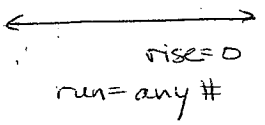


downhill
moderately steep

(E)

downhill
very steep

(F)



flat

Note: The descriptions "uphill" and "downhill" must be with the stick figure starting on the left side of the line, walking to the right.

Slope is a number used to describe how a line looks
- slope is positive if line goes uphill
- negative if downhill.
- slope is a bigger # for steep lines, smaller for less steep.
- slope is abbreviated by the letter m.

(2) Calculate slope

$$m = \frac{\text{rise}}{\text{run}}$$

(A) $m = \frac{\text{rise}}{\text{run}} = \frac{+2}{+4} = \frac{1}{2}$

(B) $m = \frac{\text{rise}}{\text{run}} = \frac{+4}{+4} = 1$

(C) $m = \frac{\text{rise}}{\text{run}} = \frac{4}{2} = 2$

(D) $m = \frac{\text{rise}}{\text{run}} = \frac{\text{any}}{0} = \text{undefined.}$

* vertical lines have undefined slope

(E) $m = \frac{\text{rise}}{\text{run}} = \frac{-2}{+4} = -\frac{1}{2}$

(F) $m = \frac{\text{rise}}{\text{run}} = \frac{-4}{4} = -1$

(G) $m = \frac{\text{rise}}{\text{run}} = \frac{-4}{2} = -2$

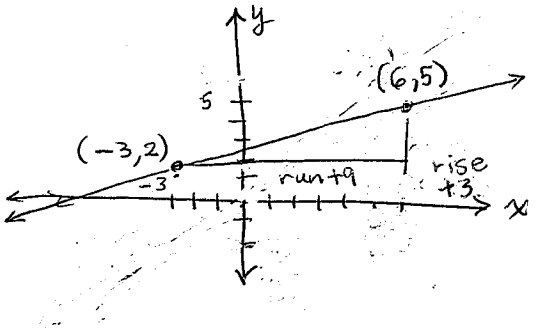
(H) $m = \frac{\text{rise}}{\text{run}} = \frac{0}{\text{any}} = 0$

* horizontal lines have slope zero

(3) Measure the rise and the run between the points $(-3, +2)$ and $(6, 5)$.

Exploring (the long way)

Graph the two points and draw the line connecting them



Draw a vertical line and a horizontal line below the line to form a $\frac{\text{rise}}{\text{run}}$ triangle.

(always a right triangle.)

Line goes uphill — slope should be positive.

Measure the length of rise and run. and calculate slope

$$m = \frac{\text{rise}}{\text{run}} = \frac{3}{9} = \boxed{\frac{1}{3}}$$

CAUTION: Slope is always $\frac{\text{rise}}{\text{run}}$ — never "run" in numerator

Question: Which variable (x or y) should we use for finding the "rise"?

Answer: The y-variables tell us up/down movement, or rise.

Question: What operation $(+, -, \times, \div)$ should we use to get rise = 3 from y-coordinates 5 and 2?

Answer: Subtract. $5 - 2 = 3$.

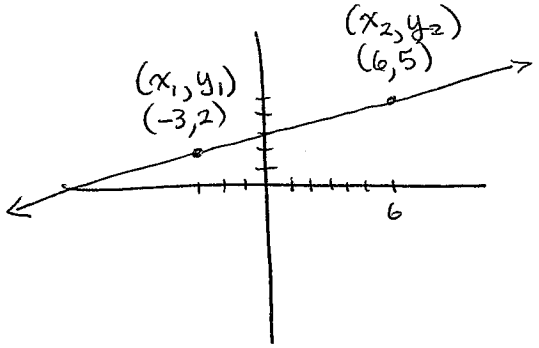
Question: Which variable should we use to find "run"?

Answer: x-variables tell us left/right or run.

Question: What operation should we use to get run = 9 from x-coordinates 6 and -3 ?

Answer: Subtract. $6 - (-3)$
$= 6 + 3$
$= 9$.

Slope Formula (the short way)



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{5 - 2}{6 - (-3)}$$
$$= \frac{3}{9}$$
$$= \boxed{\frac{1}{3}}$$

No graph needed!!

CAUTION: Decide and label the ordered pairs as (x_1, y_1) and (x_2, y_2) and do not change your mind. or mix them up.

(4) Find the slope of the line connecting $(-1, 4)$ and $(5, -3)$. and interpret.

Method 1: choose $(-1, 4)$ and $(5, -3)$
(x_1, y_1) (x_2, y_2).

write formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

substitute 4 values $m = \dfrac{-3 - 4}{5 - (-1)}$

simplify $= \boxed{\dfrac{-7}{6}}$

as x increases 6 units, y decreases 7 units

Method 2: choose $(-1, 4)$ and $(5, -3)$
(x_2, y_2) (x_1, y_1)

write formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

substitute 4 values $m = \dfrac{4 - (-3)}{-1 - 5}$

$m = \dfrac{7}{-6} = \boxed{\dfrac{-7}{6}}$

or

as x decreases 6 units
y increases 7 units

(5) Find the slope of the line passing through $(-1, -5)$ and $(2, 1)$. Interpret the result.

$(-1, -5)$ $(2, 1)$
(x_1, y_1) (x_2, y_2)

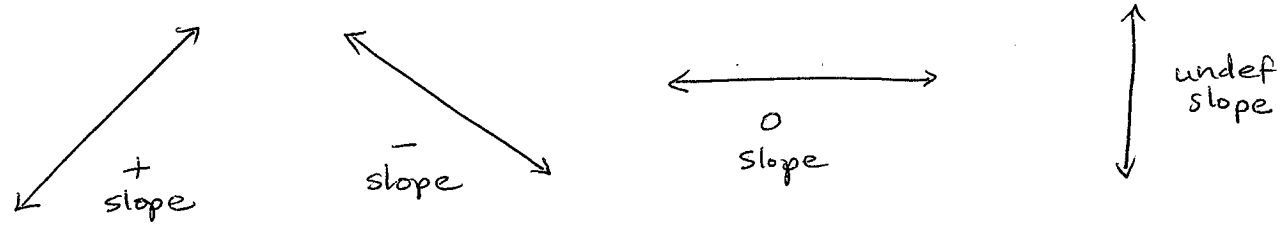
$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1 - (-5)}{2 - (-1)} = \dfrac{1+5}{2+1} = \dfrac{6}{3} = \boxed{2}$

line goes uphill

As x increases 1 unit, y increases 2 units.

CAUTION: The formula always has y in the numerator because y-variables tell us up-down or rise.

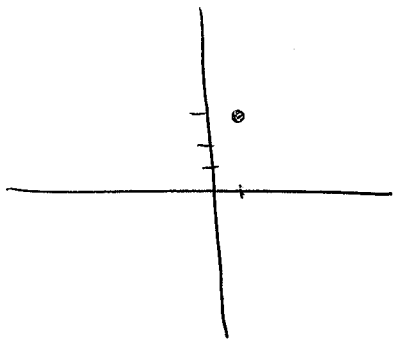
In Summary:



Note: Undefined slope is sometimes called "no slope" or "infinite slope".

(6) Graph a line with slope 2 that passes through (1,3).

step 1: Draw a graph and plot the given point.

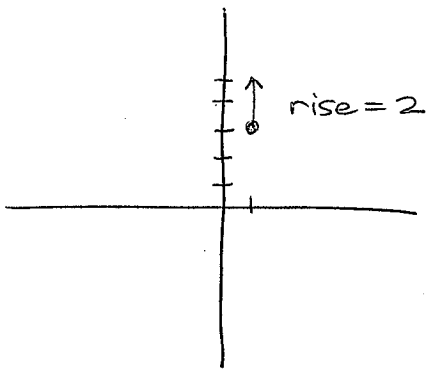


This is the starting place.

step 2: Write the slope as a fraction, and identify the rise and run

$m = \frac{2}{1}$ rise = 2, run = 1

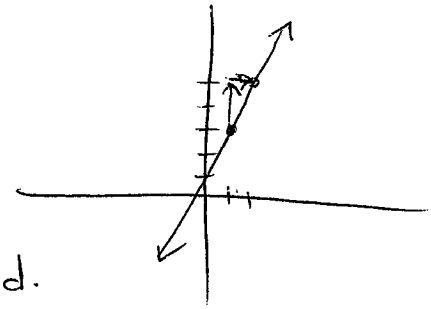
step 3: On the graph, start at the given point and go
up if rise is positive
down if rise is negative.



step 4: On the graph, start where the arrow ends and go
right if run is positive
left if run is negative.

step 5: Plot a second point and connect dots.

step 6: Repeat up to edge of grid.



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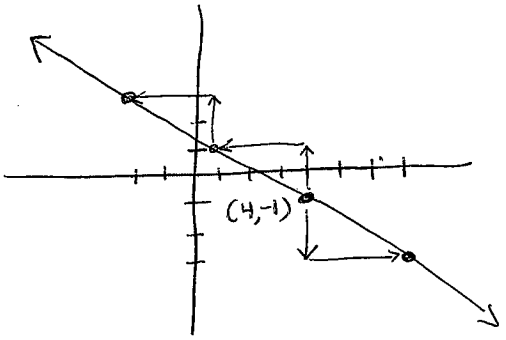
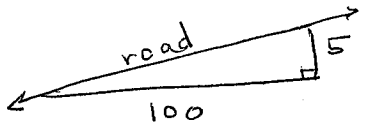
(7) Graph a line with slope $-\frac{2}{3}$ that passes through $(4,-1)$.

$m = \frac{-2}{3} \Rightarrow$ rise $= -2$, run $= 3$ OR $m = \frac{2}{-3} \Rightarrow$ rise $= 2$, run $= -3$.



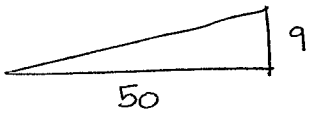
(8) The road has a 5% grade. What is its slope?



$5\% = \frac{5}{100} = \boxed{\frac{1}{20} = m}$ or $5\% = .05 = \frac{5}{100} = \boxed{\frac{1}{20}}$

Hint: Write % as decimal or directly to fraction.

(9) A road rises 9 feet for every 50 feet of horizontal distance covered. What is the grade of the road?



$m = \frac{9}{50} = 0.18 = \boxed{18\% \text{ grade}}$

Hint: To get grade (%) convert fraction to decimal to percent.

(10) A new 3-bedroom house with 1828 sq.ft. costs \$280,000.
A new 3-bedroom house with 1987 sq. ft costs. \$296,000.

Find and interpret slope of line connecting two points:
(1828, 280000) and (1987, 296000). Round to nearest hundredth.
(x_1, y_1) (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{296000 - 280000}{1987 - 1828} \ \frac{\$}{\text{sqft}}$$

$$= \frac{\$16000}{159 \text{ sqft}}$$

$$\approx \boxed{\$100.63/\text{sqft}}$$

The price increases about \$100.63 for each additional square foot

(11)

3.3.35 **Find and interpret the slope of the line containing the given points.**

$\left(\frac{1}{4}, \frac{1}{2}\right)$ and $\left(\frac{3}{8}, \frac{3}{4}\right)$

Select the correct choice below and fill in any answer boxes within your choice.

○A. m = ■ (Type an integer or a simplified fraction.)

○B. The slope is undefined.

Interpret the slope.

○A. The slope is undefined. ← if vertical line

○B. As x increases, y increases. ← slope is positive

○C. There is no change in y as x increases. ← if horizontal line

○D. As x increases, y decreases. ← slope is negative

Label the points (x_1, y_1) and (x_2, y_2)

$\left(\frac{1}{4}, \frac{1}{2}\right)$ and $\left(\frac{3}{8}, \frac{3}{4}\right)$

Plug into slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{(3/4 - 1/2)}{(3/8 - 1/4)}$$

Take a deep breath and subtract fractions.

$$= \frac{(3/4 - 2/4)}{(3/8 - 2/8)} \qquad CD = 4 \quad \frac{1}{2}\cdot\frac{2}{2} = \frac{2}{4} \qquad CD = 8 \quad \frac{1}{4}\cdot\frac{2}{2} = \frac{2}{8}$$

$$= \frac{(1/4)}{(1/8)}$$

Divide fractions

$$= 1/4 \div 1/8$$

$$= 1/4 \cdot 8/1 = 8/4 = \boxed{2}$$

(Positive ⇒ choose option B)

(12) a) Find the slope of the line passing through (1,5) and (−3,5).

(1,5) and (−3,5)
(x_1, y_1) (x_2, y_2)

Decide which is (x_1, y_1) and which is (x_2, y_2)

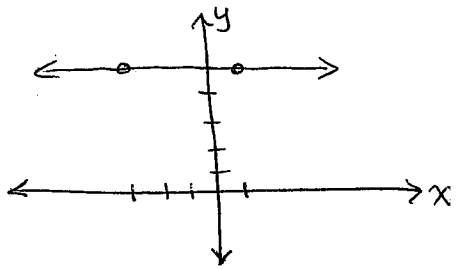
$m = \frac{y_2 - y_1}{x_2 - x_1}$ Write formula

$m = \frac{5-5}{-3-1}$ Substitute 4 values.

$= \frac{0}{-4}$

$= \boxed{0}$ Simplify

b) Graph the line in part a) and write a sentence describing what kind of lines have m=0 (zero slope).



Interpret: There is no change in y as x increases

Horizontal lines have zero slope.

Note: The points have the same y-coordinate

(13) a) Find the slope of the line passing through (2,6) and (2,−1).

(2,6) and (2,−1)
(x_1, y_1) (x_2, y_2)

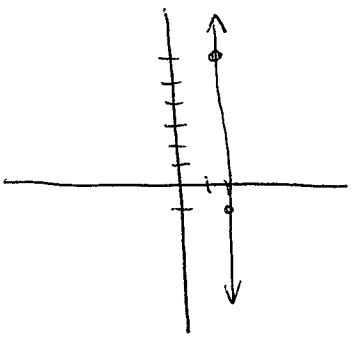
Decide.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Formula.

$= \frac{-1-6}{2-2}$

$= \frac{-7}{0} =$ undefined

b) Graph the line in part a) and write a sentence describing what kind of lines have undefined slope.



Vertical lines have undefined slope.

Note: The points have the same x-coordinate.